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Pulsating strings on $AdS_5 \times S^5$

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Abstract: We find the anomalous dimension and the conserved charges of an R-charged string pulsating on AdS_5. The analysis is performed both on the gauge and string side, where we find agreement at the one-loop level. Furthermore, the solution is shown to be related by analytic continuation to a string which is pulsating on S^5, thus providing an example of the close relationship between the respective isometry groups.

Keywords: AdS-CFT and dS-CFT Correspondence, Integrable Equations in Physics.

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Contents

1. Introduction

The AdS/CFT conjecture [1, 2, 3] has lead to a better understanding of both conformal gauge theories as well as string theory in curved spaces. Within this framework, the seminal work of [4] included a discussion of operators of the form $\mathrm{Tr}\, Z^{J_1} W^{J_2} + \cdots$ (built up from the scalars Z and W of the $\mathcal{N} = 4$ SYM supermultiplet) where $J_1 \ll J_2$. The dots indicate other permutations of the fields Z and W inside the trace, and in general these states mix among themselves under scaling; only certain linear combinations are eigenstates to the scaling operator. Semiclassical string configurations which usually go beyond the BMN limit (e.g. by taking both J_1 and J_2 to be large) have since been studied extensively [5]–[22]), and are reviewed in [23].

The observation of [24] that the matrix of anomalous dimensions could be mapped to an integrable Bethe spin chain [25] simplified and extended the studies of the corresponding gauge theory [26]–[37] (see also [38, 39]), reviewed in [40]. The original results of [24] were restricted to the group SO(6) at 1-loop level, but were later extended to the full 1-loop $SU(2, 2|4)$ chain [41, 42], taking advantage of previous results on integrability in QCD amplitudes [43, 44] and the QCD dilatation operator [45, 46, 47] (see also [48]–[52]). Progress on higher orders in closed subsectors has also been made [53]–[57].

The integrable spin chain formulation exposes the conserved charges. Conserved charges in the sigma model were first discussed in [58, 59] (see also [60, 61]). Progress on relating the conserved charges on either side to each other by viewing (subsectors of) both sides of the duality as an integrable system was made in [62]–[65] (see also [66]). The work on finding descriptions of the AdS/CFT duality in terms of integrable systems are reviewed in [67].

In this paper, we will analyse a string pulsating on AdS_5 and whose centre of mass is revolving on S^5, both from the gauge and string side (assuming large quantum numbers). From the AdS/CFT conjecture, we expect that the anomalous dimension of the corresponding operator will coincide with the first order energy correction on the string side. Another motivation for studying this configuration is that the conserved charges on either side of the duality can be matched explicitly using integrability.

A third motivation is that our solution will be shown to be related by an analytic continuation to the solution of [31] for a string pulsating and revolving on S^5. This provides an example of the close mathematical relationship between the isometry groups of AdS_5 and S^5; SO(4, 2) and SO(6), respectively. Such relations were discussed in [29], where a first example was given; a string rotating in two planes on S^5 was shown to be related by analytic continuation to a string whose centre of mass is revolving in one plane on S^5 and with one spin in AdS_5. One may speculate that tying together seemingly different solutions in this way may help in providing a bridge between duality checks at the level of individual solutions and higher-level checks. An example of the latter is the recent analysis of the duality at the level of actions [68]–[73].

We will analyse the case at hand from the gauge side and string side in sections 2 and 3, respectively. In section 4 we exhibit the conserved charges on the string side. Our conclusions are presented in section 5.

2. Gauge side

In this section, we will consider operators of the form $\mathrm{Tr}(D\bar{D})^B Z^J$, which are charged under SO(2, 2). Here, $D \equiv D_1 + iD_2$, (where D_i are covariant derivatives) and Z is one of the three complex scalars of the $\mathcal{N} = 4$ supermultiplet. Individual operators are formed by linear combinations of different distributions of the D's and $\bar{D}$'s over the Z's. In general mixing occurs under scaling within the full SO(4, 2). However, in the semiclassical limit it turns out that it *will* be sufficient to consider the bosonic subgroup SO(2, 2) [57], cf. what happens in the SO(6) case [31, 65]. The mapping of the matrix of anomalous dimensions to a hamiltonian of a spin chain will then allow us to find the the eigenvalues of the diagonalized system.

The simple roots of SO(2, 2) are $\vec{\alpha}_1 = (1, 1)$ and $\vec{\alpha}_2 = (1, -1)$. In the infinite-dimensional representation of highest weight $\vec{w} = (-1, 0)$, the Bethe equations are

$$\left(\frac{u_{q,i} + i\vec{\alpha}_q \cdot \vec{w}/2}{u_{q,i} - i\vec{\alpha}_q \cdot \vec{w}/2}\right)^L = \prod_{j\neq i}^{n_q} \frac{u_{q,i} - u_{q,j} + i\vec{\alpha}_q \cdot \vec{\alpha}_q/2}{u_{q,i} - u_{q,j} - i\vec{\alpha}_q \cdot \vec{\alpha}_q/2} \prod_{q'\neq q} \prod_{j}^{n_{q'}} \frac{u_{q,i} - u_{q',j} + i\vec{\alpha}_q \cdot \vec{\alpha}'_q/2}{u_{q,i} - u_{q',j} - i\vec{\alpha}_q \cdot \vec{\alpha}'_q/2}, \tag{2.1}$$

as written in [24] for an arbitrary Lie group, and

$$\begin{aligned} \left(\frac{u_{1,i} - i/2}{u_{1,i} + i/2}\right)^L &= \prod_{j\neq i}^{n_1} \frac{u_{1,i} - u_{1,j} + i}{u_{1,i} - u_{1,j} - i} \\ \left(\frac{u_{2,i} - i/2}{u_{2,i} + i/2}\right)^L &= \prod_{j\neq i}^{n_2} \frac{u_{2,i} - u_{2,j} + i}{u_{2,i} - u_{2,j} - i} \end{aligned} \tag{2.2}$$

for SO(2, 2). The anomalous dimension is

$$\gamma = \frac{\lambda}{8\pi^2}\left(\sum_{i=1}^{n_1}\frac{1}{u_{1,i}^2+1/4}+\sum_{i=1}^{n_2}\frac{1}{u_{2,i}^2+1/4}\right). \tag{2.3}$$

As indicated, there are n_q roots of the type q. The form of the operator we are looking for is $\mathrm{Tr}(D\bar{D})^B Z^J$, so the number of sites is $L = J$. The two root types essentially correspond to creation of D's and $\bar{D}$'s, respectively, so we set $n \equiv n_1 = n_2 = B$.

Assuming that the number of roots is large (so that they can be approximated by a continuous distribution) in the thermodynamic limit (i.e. a large number of sites L) the log of the Bethe equation for the first type of root (after a rescaling $u \to uL$) is

$$\frac{2}{\alpha}\left(-\frac{1}{u}+2\pi m\right) = 2\,⨍_C \frac{\sigma(u')du'}{u-u'}\,, \tag{2.4}$$

where the line through the integral sign indicates that the singularity at $u' = u$ is resolved by taking the principal value of the integral. The contour C is defined by the support of the root density $\sigma(u')$ and its endpoints are a and b. We have defined $\alpha \equiv n/L$. The integer m corresponds to different branches of the log. The root density is normalized as

$$\int_C \sigma(u')du' = 2\,. \tag{2.5}$$

Reading (2.4) as a force balancing equation, we conclude that the roots are repelled from each other but attracted to the point $u = 1/2\pi m$. We therefore expect that the roots will spread out along the contour C passing through this point.

Performing an inverse Hilbert transform on (2.4), the root density is

$$\sigma(u) = -\frac{1}{\pi^2\alpha}\left[(u-a)(u-b)\right]^{1/2} ⨍_C du'\left(\frac{1}{u'}-2\pi m\right)\frac{1}{u'-u}\frac{1}{\left[(u'-a)(u'-b)\right]^{1/2}}\,. \tag{2.6}$$

The multivalued function $[\cdots]^{1/2}$ has a cut along the segment of the real axis coinciding with the contour C. Calculating the integral by deforming the contour and picking up the residue at $u' = 0$, we get

$$\sigma(u) = -\frac{i}{\pi\alpha u\sqrt{ab}}\left[(u-a)(u-b)\right]^{1/2}. \tag{2.7}$$

The endpoints a and b of the contour C are determined by inserting (2.7) into equations (2.4) and (2.5). This results in the two equations

$$\begin{aligned}\sqrt{ab} &= \frac{1}{2\pi m}\\ a+b &= \frac{1+2\alpha}{\pi m}\,.\end{aligned} \tag{2.8}$$

In particular, this means that for non-negative (i.e. physical) values of α, the endpoints of the contour will lie on the positive real axis (for positive m) and the contour will pass through the point $u = 1/2\pi m$, as expected.

Now define the resolvent

$$W(u') \equiv \int_C du \frac{\sigma(u)}{u' - u} \,. \tag{2.9}$$

By deforming the contour and picking up residues at $u = 0$, $u = u'$ and $u = \infty$, the resolvent becomes [31, 63]

$$-\alpha W(u') = \frac{1}{u'} \left[1 - \sqrt{(1 - 2\pi m u')^2 - 2\alpha(4\pi m u')} \right] + \pi m \,. \tag{2.10}$$

The square root denotes the branch which coincides with the principal branch for small u'. One of the virtues of the resolvent is that it determines the first part of the anomalous dimension (2.3) in the thermodynamic limit:

$$\gamma_1 = -\frac{\lambda\alpha}{16\pi^2 L} W'(0) \,. \tag{2.11}$$

Inserting (2.10) into (2.11), we get

$$\gamma_1 = +\frac{\lambda m^2}{2L} \alpha(1 + \alpha) \,. \tag{2.12}$$

According to (2.2), the two root types behave symmetrically and do not interact. Due to the trace condition, which in this case takes the form

$$\prod_{j=1}^{j=L} \frac{(u_{1,j} + i/2)(u_{2,j} + i/2)}{(u_{1,j} - i/2)(u_{2,j} - i/2)} = 1 \,, \tag{2.13}$$

the second type of roots spread out along along the segment $[-b, -a]$ of the negative real axis, so (2.3) becomes

$$\gamma = \gamma_1 + \gamma_2 = 2\gamma_1 = +\frac{\lambda m^2}{L} \alpha(1 + \alpha) = +\frac{\lambda m^2 B}{J^2} \left(1 + \frac{B}{J} \right) . \tag{2.14}$$

The conformal dimension is $\Delta = 2B + J$, in terms of which

$$\gamma = m^2 \lambda \frac{\Delta^2 - J^2}{4J^3} \,. \tag{2.15}$$

3. String side

In [8], a string pulsating on AdS_5 or S^5 was considered. In [31], the latter configuration was generalized to include a rotation in one plane on S^5. In this section, we will consider the closely related sigma model description of a string pulsating on AdS_5 and whose centre of mass is revolving on S^5. Restricting the motion to the subspace $AdS_3 \times S^1$ means that the isometry group contains a factor isomorphic to SO(2, 2), hence matching the set of operators considered in section 2.

The metric on $AdS_5 \times S^5$ will be written

$$\begin{aligned} ds^2_{AdS_5} &= d\rho^2 - \cosh^2 \rho dt^2 + \sinh^2 \rho (d\theta^2 + \cos^2 \theta d\Phi_1^2 + \sin^2 \theta d\Phi_2^2) \\ ds^2_{S^5} &= d\gamma^2 + \cos^2 \gamma d\phi_3^2 + \sin^2 \gamma (d\Psi^2 + \cos^2 \Psi d\phi_1^2 + \sin^2 \Psi d\phi_2^2)\,. \end{aligned} \tag{3.1}$$

We will use the notation $\phi \equiv \phi_3$. The relevant metric for the $AdS_3 \times S^1$ subspace of the full space is

$$ds^2 = d\phi^2 + d\rho^2 - \cosh^2 \rho dt^2 + \sinh^2 \rho d\theta^2\,. \tag{3.2}$$

We assume that the string is wrapped around the azimuthal angle θ on AdS_3. We then use the ansatz

$$\phi = \phi(\tau)\,, \qquad \rho = \rho(\tau)\,, \qquad t = \tau\,, \qquad \theta = m\sigma\,. \tag{3.3}$$

The integer m allows for multi-wrapping. We will consider t and θ to be gauge fixed. The Nambu-Goto action is

$$S = -m\sqrt{\lambda} \int dt \sinh \rho \sqrt{\cosh^2 \rho - \dot{\phi}^2 - \dot{\rho}^2}\,. \tag{3.4}$$

The energy $\pi_t \propto H$ and the spin π_ϕ are conserved. The dynamical momenta are

$$\pi_\rho = \frac{m\sqrt{\lambda}\dot{\rho} \sinh \rho}{\sqrt{\cosh^2 \rho - \dot{\phi}^2 - \dot{\rho}^2}} \tag{3.5}$$

and

$$\pi_\phi = \frac{m\sqrt{\lambda}\dot{\phi} \sinh \rho}{\sqrt{\cosh^2 \rho - \dot{\phi}^2 - \dot{\rho}^2}}\,. \tag{3.6}$$

The hamiltonian is then given by

$$H^2 = (\pi_\rho \dot{\rho} + \pi_\phi \dot{\phi} - L)^2 = \cosh^2 \rho (\pi_\rho^2 + \pi_\phi^2 + m^2 \lambda \sinh^2 \rho)\,. \tag{3.7}$$

Following [31], we now consider the term $V(\rho) = m^2 \lambda \cosh^2 \rho \sinh^2 \rho$ to be a perturbation. A hermitean form of the unperturbed hamiltonian operator acting on a wave function is then $\hat{H}_0^2 \Psi(\rho) = \cosh \rho (\hat{\pi}_\rho^2 + \hat{\pi}_\phi^2) \cosh \rho \Psi(\rho)$, i.e.

$$\Delta^2 \Psi(\rho) = -(\cosh \rho) \nabla^2 (\cosh \rho) \Psi(\rho) + J(J+4) \cosh^2 \rho \Psi(\rho), \tag{3.8}$$

where

$$\nabla^2 = \frac{1}{\sinh^3 \rho \cosh \rho} \frac{\partial}{\partial \rho} \sinh^3 \rho \cosh \rho \frac{\partial}{\partial \rho}\,. \tag{3.9}$$

J and Δ are non-negative integers. Introducing $x = \frac{1}{\cosh^2 \rho}$ and restricting to even integers $J = 2j$, $\Delta = 2a$ transforms (3.8) into

$$-\frac{x^{7/2}}{1-x} \frac{d}{dx} \frac{(1-x)^2}{x} \frac{d}{dx} \frac{1}{x^{1/2}} \Psi(x) + j(j+2)\Psi(x) - a^2 \Psi(x) = 0\,. \tag{3.10}$$

The power series ansatz

$$\Psi(x) = \sum_{\lambda=0}^{\lambda=\infty} a_\lambda x^{k+\lambda} \tag{3.11}$$

results in an indicial equation with two solutions. The solution which keeps all terms finite on the interval $0 \leq x \leq 1$ is $k = j + 5/2$. Hence, the recursion relation becomes

$$a_\lambda = -a_{\lambda-1}\frac{(a+1+\lambda+j)(a-1-\lambda-j)}{\lambda(\lambda+2+2j)}\,, \tag{3.12}$$

whose solution is

$$a_p = (-1)^p \binom{a-j-2}{p} \frac{(a+j+1+p)!}{(p+2j+2)!}\,. \tag{3.13}$$

Inserting this into (3.11) gives the wave functions, whose normalized forms are

$$\Psi(x) = \frac{2\sqrt{a(a-j-1)}}{(a-j-1)!\sqrt{a+j+1}}\frac{1}{x^{j-1/2}}\left(\frac{d}{dx}\right)^{a-j-1} x^{a+j+1}(1-x)^{a-j-2}\,. \tag{3.14}$$

The first order correction to the energy is

$$\Delta E^2 = \int d\tau \Psi(x)V(x)\Psi(x) = \frac{2a(a+j+1)(a-j-1)}{(j+1)(2j+1)(2j+3)}\,, \tag{3.15}$$

where $d\tau$ is the volume element. The energy to first order and for large quantum numbers is then $E = \Delta + \gamma$, where

$$\gamma = m^2\lambda\frac{\Delta^2 - J^2}{4J^3}\,. \tag{3.16}$$

This agrees with the expected anomalous dimension (2.15).

4. Conserved charges

On the gauge side, the mapping of the matrix of anomalous dimensions to a hamiltonian of an integrable spin chain immediately provides all conserved charges in terms of the resolvent. Following [63], the recent paper [64] demonstrates that for the R-charge assignment (J_1, J_2, J_2) on S^5, the charges on the string side precisely match those on the gauge side (at the 1-loop level). In our case, the 1-loop resolvent is (2.10). On the string side, the corresponding generator is essentially given by the quasi-momentum, as discussed in [65]. In this section, we will follow the procedure outlined in [65] to exhibit the quasi-momentum for the case at hand.

We are assuming that the string is moving on an $AdS_3 \times S^1$ subspace. The AdS_3 space can be described as the hypersurface $-X_1^2 - X_2^2 + X_3^2 + X_4^2 = 1$ in $\mathbb{R}^4$. Defining $W \equiv X_1 + iX_2$ and $Z \equiv X_3 + iX_4$, this space can be equivalently described as an $\mathrm{SU}(1,1)$ group manifold using the map

$$\begin{pmatrix} Z & W \\ \bar{W} & \bar{Z} \end{pmatrix} = g \in \mathrm{SU}(1,1)\,. \tag{4.1}$$

As an ansatz for the string pulsating on AdS_3 and revolving on S^1 we use

$$\begin{aligned} W &= \sinh\rho e^{i\theta} \\ Z &= \cosh\rho e^{it} \end{aligned} \tag{4.2}$$

In this section we will use the Polyakov action in unit gauge,

$$\begin{aligned} S &= \frac{\sqrt{\lambda}}{4\pi}\int d\sigma d\tau \left[\partial Z\partial\bar{Z} - \partial W\partial\bar{W} - (\partial X_5)^2\right] \\ &= -\frac{\sqrt{\lambda}}{4\pi}\int d\sigma d\tau \left[\frac{1}{2}\operatorname{Tr}(g^{-1}\partial_\alpha g)^2 + (\partial X_5)^2\right] \end{aligned} \tag{4.3}$$

(hence choosing X_3 and X_4 to be time-like). In this description, we will no longer consider $t(\tau)$ to be gauge fixed. By the equations of motion, $\phi \equiv X_5 = Q\tau$. The action is invariant under constant shifts along the circle, so $Q\sqrt{\lambda} \equiv J$ is the conserved charge corresponding to the spin.

The action (4.3) is also invariant under constant left and right shifts of the group elements, $g \to hg$ and $g \to gh$. The corresponding charge is the energy

$$E = +\sqrt{\lambda}Q_l = -\sqrt{\lambda}Q_r = -\sqrt{\lambda}\dot{t}\cosh^2\rho\,. \tag{4.4}$$

In the following, we will restrict our considerations to the time τ when $\rho(\tau) = t(\tau) = 0$. Then

$$\frac{E^2}{\lambda} = \dot{t}^2 = \dot{\rho}^2 + Q^2\,, \tag{4.5}$$

where the last equality follows from the constraint corresponding to fixing the gauge in the Polyakov action.

Defining $\partial_\pm \equiv \partial_\tau \pm \partial_\sigma$ and currents $j_\pm \equiv g^{-1}\partial_\pm g$, it follows from the constraint $Z\bar{Z} - W\bar{W} = \det(g) = 1$ that

$$0 = \partial_+ j_- - \partial_- j_+ + [j_+, j_-]\,. \tag{4.6}$$

This coincides with the consistency condition $[L, M] = 0$ for the linear problem $L\Psi = M\Psi = 0$, where

$$\begin{aligned} L &= \partial_\sigma + \frac{1}{2}\left(\frac{j_+}{1-x} - \frac{j_-}{1+x}\right) \\ M &= \partial_\tau + \frac{1}{2}\left(\frac{j_+}{1-x} + \frac{j_-}{1+x}\right)\,. \end{aligned} \tag{4.7}$$

Explicitly, the first equation is

$$\partial_\sigma\Psi = \frac{x}{x^2-1}\begin{pmatrix} iQ_l & \dot{\rho}e^{im\sigma} \\ \dot{\rho}e^{-im\sigma} & -iQ_l \end{pmatrix}\Psi\,. \tag{4.8}$$

Considering Ψ to be a vector of the type

$$\Psi = \begin{pmatrix} Ae^{ip_+\sigma/2\pi} \\ Be^{ip_-\sigma/2\pi} \end{pmatrix} \tag{4.9}$$

provides a family of solutions[1] to (4.8), provided that the condition

$$\left[\frac{x^2-1}{2\pi}(p_\pm \mp \pi m)\right]^2 + (x\dot\rho)^2 = \left[\frac{m(x^2-1)}{2} - xQ_l\right]^2 \tag{4.10}$$

is satisfied. Consequently, each root $p_\pm(x)$ will be double-valued. Subtracting the poles from one of the sheets of p_- (with a branch cut along the positive real axis), the resolvent is

$$G(x) = \frac{2\pi}{x^2-1}\left[-xQ + \left\{\left[\frac{m}{2}(x^2-1) - xQ_l\right]^2 - (x\dot\rho)^2\right\}^{1/2}\right] - \pi m\,. \tag{4.11}$$

Rescaling $x \to 4\pi Q x$, the leading contribution for large quantum numbers ($\lambda \to 0$ in $Q = J/\sqrt{\lambda}$ and $Q_l = E/\sqrt{\lambda}$) is

$$-G_0(x) = \frac{1}{2x}\left[1 - \sqrt{(1-2\pi m x)^2 - 2\alpha(4\pi m x)}\right] + \pi m\,. \tag{4.12}$$

The square root denotes the branch which coincides with the principal branch for small x. It is proportional to the resolvent (2.10) on the gauge theory side. Since the charges are generated by the odd part of the resolvent, this shows that the charges on the gauge and string side match.

5. Conclusions

We considered a string pulsating on AdS_5 and revolving on S^5. The anomalous dimension (2.15) agrees with the first order energy correction (3.16), as expected from the AdS/CFT conjecture. In terms of $\alpha \equiv \frac{n}{J} = \frac{\Delta - J}{2J}$, these results become

$$\gamma = m^2\lambda\frac{1}{J}\alpha(1+\alpha)\,. \tag{5.1}$$

Consider analytically continuing this result to the unphysical region $\alpha < 0$ by $\Delta \to -J_1$ and $J \to -L$. This takes $\alpha \to -\alpha_{EMZ} \equiv \frac{J_1 - L}{2L}$, i.e.

$$\gamma \to m^2\lambda\frac{1}{L}\alpha_{EMZ}(1-\alpha_{EMZ})\,. \tag{5.2}$$

This is the result[2] of [31] for a string pulsating and revolving on S^5, which together with our result provides a complete description for all real values of α; for $\alpha < 0$, the string pulsates and revolves on S^5. As α is turned to positive values,[3] the string starts pulsating on AdS_5 instead (while still revolving on S^5). On the gauge side, the corresponding operator forms are $\mathrm{Tr}(Z\bar Z)^{(L-J_1)/2}X^{J_1}$ and $\mathrm{Tr}(D\bar D)^{(\Delta-J)/2}Z^J$, respectively.

[1] The ansatz $p_\pm(x) = a(x) \pm \pi m$ is helpful.

[2] Our definition of α_{EMZ} differs by a factor of 2 from that of [31].

[3] Note that we assumed large quantum numbers. As $J \to 0$, the thermodynamic limit is no longer valid. The behaviour of the anomalous dimension in the strong coupling region is discussed in [8]. A similar phenomenon occurs in [29].

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This description can be compared to the extension of [29] to the results of [28]. In [28], operators of the form $\mathrm{Tr}\, Z^{J_1} W^{J_2}$ were considered, corresponding to strings rotating in two planes on S^5. It was shown in [29] that the replacements $J_1 + J_2 \to -J$, $J_2 \to S$ and $\gamma \to -\gamma$ turned the system of Bethe equations[4] and anomalous dimensions into a description of operators of the form $\mathrm{Tr}\, D^S Z^J$, corresponding to strings rotating both on AdS_5 and S^5.

Let us also mention that the presence of an integrable structure on both sides of the duality is manifested in our case by the agreement of the corresponding generators of conserved charges, (2.10) and (4.12). For the string pulsating on S^5, considered in [31], the corresponding check was carried out in [65].

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